

Particle Filter Based Traffic State Estimation Using Cell Phone Network Data

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Abstract— Using cell phones as traffic probes is a promising Intelligent Transportation System technology. Compared with traditional traffic data collecting approaches, cellular probe has the advantage of the ready-to-use infrastructure and the wide coverage. This paper presents two Bayesian framework based traffic estimation models by the measurement of cell handoff data of floating vehicles. The first and the simpler model uses traffic speed as the only state variable. The second-order model, incorporating traffic volume as the second state variable, has a two-level architecture, where macroscopic states and microscopic states are connected by the process of state reconstruction. This mechanism makes it possible to realize high-order sparse-sampling traffic estimation. Due to the good performance on solving highly nonlinear estimation problems, Particle Filters are introduced to provide the approximation solution of traffic state estimation problems with system noise and measurement error. The performance evaluation and practical test of Particle Filters under different data sets are performed by numerical experiments.

I. INTRODUCTION

WIRELESS communication technologies, including GPS, WI-FI and cell phone etc., have been increasingly used for ITS applications for both infrastructure-based systems and vehicle-based systems for the past few years. An especially promising approach among them is the traffic probe based on cellular network (or mobile phone network) [1][2], which has two distinct advantages from the perspective of data acquisition. One is the cost effective way of implementation. Nearly no extra expensive infrastructure needs to be installed because all it needs are almost ready. The other is the coverage over a wide area that implies the ubiquitous usage.

Basically there are two technical options to make traffic probes based on cell phone: handset-based solutions and network-based solutions [1][2]. The handset-based solutions rely on other additional technology, usually a GPS receiver. Although this approach can provide high location accuracy, a specially designed and expensive handset is needed which

will also increase the additional communication costs. On the opposite, the network-based solutions are a passive but intelligent way that only uses the data which has already taken place in the network, such as handoff time. Handoff is a mechanism that transfers the ongoing call from one cell to another when a cell phone moves through the coverage area of a cellular system [3]. Any vehicle with a cell phone may potentially act as a traffic probe and the average traffic speed can be derived from handoff data. This approach is appealing but few literatures have been published on the research of traffic estimation method using cell phone data [4].

In order to make it easy for on-line implementation, researchers prefer to focus on macroscopic traffic models which represent the average traffic behavior using aggregated variables. Bayesian frameworks [5] are usually used to cope with those non-linear models with uncertainty. According to Bayesian theory all information about the states of interest can be obtained from the posterior state distribution [5]. Because Bayesian estimation problem cannot be solved analytically in general, approximation approaches have been proposed. A recently developed, powerful and scalable approximation approach is Particle Filter or Sequential Monte Carlo Method [6] [7]. It computes the posterior density function of the state by an empirical histogram obtained by a Monte Carlo simulation. It doesn't need to be based on the linearization of the state and the observation models and the assumption of Gaussian noises. In [8], a solution of highway traffic estimation is proposed using a sequential Monte Carlo algorithm, based on first-order traffic models (only traffic density is considered). In [9], the freeway traffic is described by a second-order model where both traffic density and speed are estimated. In this paper, we first investigate a simple first-order traffic estimation model and then a two-level second-order model, partly based on the framework in [9], both of which are suitable for the traffic on freeways or urban expressways with few interchanges or intersections.

II. PROBLEM FORMULATION

The mobile phone system (or cellular network) comprises handsets (mobile stations, or cell phones) and base stations. Given a regular spacing of base stations, the boundary of the areas covered by each base station are hexagonally shaped and are termed a cell (see Figure 1). Different cell-types can be classified according to their coverage dimension [10].

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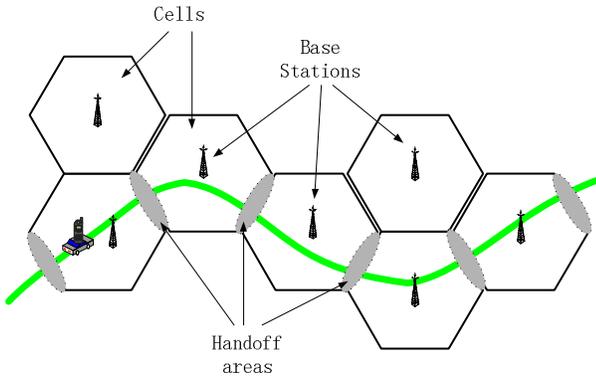


Fig. 1. Vehicle with an ongoing cell phone call traversing handoff areas

Handsets monitor the M (in GSM, $M=5$) strongest signals and report regularly the measurements to the network. Based on the measurements, the network centrally transfers connections from one base station to another (handoff) when the communicating handset is moving during the service session. We assume that only micro-cells (with dimension of 0.1-1 km) of similar size are discussed in this paper.

Most of the handoff algorithms are based on signal level measurement. A handoff will occur when the averaged signal level from new base station exceeds that from current one plus a hysteresic level [4][11].

Due to the existence of noise in signal measurement, the positions of handoff (or handoff points) float randomly according to some probabilistic distribution. But we are able to compute where the mean handoff points are if we obtain their distribution based on numerous statistical experiments.

Here we can build a simplified one-dimension traffic model based on the statistics of cell phone handoffs. Assume the object we are interested in is a segment of a freeway or an

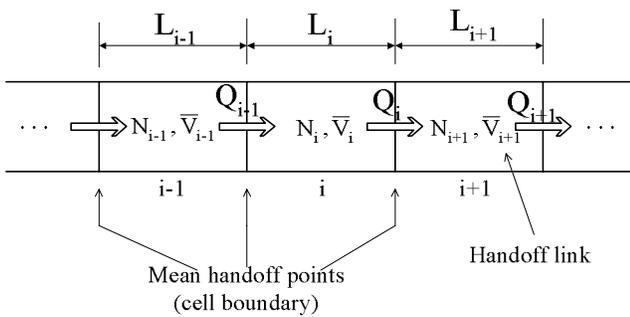


Fig. 2. Simplified one-dimension traffic model composed of handoff links

urban expressway that doesn't have many intersections and in/out ramps and is full covered by base station signals. When a vehicle with a cell phone in communication drives along the freeway, it will traverse the cell boundaries frequently and handoff operations will occur again and again. Here we concentrate our attention on the traffic flows in a one-dimension space. As it can be observed in Figure 2, the

road segment can be modeled as a straight line, divided into several smaller sections which are connected one by one and separated by the mean handoff points. A four-tuple can be used to describe the data structure of each measurement of handoffs.

$$H = (ID_{cellphone}, t_{handoff}, cell_{from}, cell_{to}) \quad (1)$$

We call the two consecutive handoffs of the same cell phone a handoff pair.

III. TRAFFIC MODEL

The traffic flow in this paper is modeled as a stochastic dynamic system with discrete-time states. Usually we choose the average traffic speed and the number of vehicles in handoff links (in short, links, in the following part of the paper) as global state variables. For link i , $i=1, \dots, n$, the state is $x_{i,k} = \{N_{i,k}, \bar{v}_{i,k}\}^T$, where $N_{i,k}$ is the number of vehicles in section i at sampling time t_k , and $\bar{v}_{i,k}$ is their average speed. Here state vector $x_k = \{x_{1,k}^T, x_{2,k}^T, \dots, x_{n,k}^T\}$ is sampled at time $t_1 < t_2 < \dots < t_k < \dots$.

The generic model of system state evolution of discrete time estimation problem is as follows

$$x_{k+1} = f_k(x_k, w_k) \quad (2)$$

where f_k is the system transition function and w_k , the system noise, is a zero mean white-noise sequence independent of past and current states. The probability density function (PDF) of w_k is assumed to be known. The measurement y_k is related to the states via observation equation

$$y_k = h_k(x_k, \eta_k) \quad (3)$$

where h_k is the measurement function and η_k , the measurement error, is another zero mean white-noise sequence of known PDF, independent of past and present states and system noise.

A. First-order Traffic Model

For simplicity we consider traffic speed as the only state variable here. The system model can be written as

$$\bar{v}_{i,k+1} = \alpha_{i,k} \bar{v}_{i-1,k} + \beta_{i,k} \bar{v}_{i,k} + \gamma_{i,k} \bar{v}_{i+1,k} + w_{i,k} \quad (4)$$

$$y_{i,k} = v_{i,k}^{avg} + \eta_k \quad i = 1, 2, \dots, n \quad (5)$$

where

$$v_{i,k}^{avg} = \sum_{j=1}^{M_{i,k}} L_i / (t_j^+ - t_j^-) \quad (6)$$

is the average measured traffic speed in time period k .

$M_{i,k}$ is the amount of observed handoff pairs about link i in time period k . L_i is the length of link i between two mean handoff points. t_j^- is the observed first handoff time from

link $i-1$ to link i and t_j^+ is the observed second handoff time from link i to link $i+1$, $t_j^+ > t_j^-$ and both of them buildup the handoff pair j . $\alpha_{i,k}$, $\beta_{i,k}$, and $\gamma_{i,k}$ are coefficients used to weigh how the average traffic speed in neighboring links has an impact on the average speed in current link.

While road traffic is in a stable pattern, we can set all coefficients $\alpha_{i,k}$, $\beta_{i,k}$, $\gamma_{i,k}$ as constants α , β , γ , or let them satisfy (but not necessarily) $\alpha_{i,k} + \beta_{i,k} + \gamma_{i,k} = 1$.

From (4) and (5), we see that the first-order traffic model is linear so that either a Kalman Filter or a Particle Filter is able to be built in order to estimate the system states without difficulty.

B. Second-order Traffic Model

If we incorporate traffic volume as the second state variable, we can build a second-order traffic model. There are two options to choose the second state variable: the number of vehicles $N_{i,k}$, present in link i at sample k , or $Q_{i,k}$, the number of vehicles crossing the cell boundary from link i to link $i+1$, during the time interval k . According to the definition, these two variables always satisfy $N_{i,k+1} = N_{i,k} + Q_{i-1,k} - Q_{i,k}$, for all i,k . Hence each of them can be transformed into the other despite which of them will be chosen.

When we use cell phones as traffic probe, the frequency of appearance of valid raw data is much lower than using loop detector data which is a main difficulty for estimation. One of the reasons is that for safety motives, not everyone is willing to use a cell phone when driving and also only a small part of passengers will get an incoming call when they are on the road. Another reason is that we only hope an ongoing cell phone call lasting long enough to traverse two continuous cell borders, i.e. constructing a handoff pair, to get a set of valid data. Obviously, handoff data satisfying such conditions is sparse. Besides, the underlying geographic uncertainty between road networks and cellular networks causes the locations of handoff points and the lengths of handoff links to be very irregular. As a result, missing data is a common problem during data pre-processing. We call traffic estimation based on cellular probe as sparse-sampling traffic estimation and call traffic estimation based on loop detection as dense-sampling traffic estimation. Next we will present a two-level second-order traffic model which is suitable for sparse-sampling traffic estimation.

In the upper level, or macroscopic level, system state equations are as follows,

$$Q_{i,k+1} = \sum_{t=1}^{\tau_k} U_{i,t} + w_{i,k}^1 \quad (7)$$

$$V_{i,k+1} = \frac{1}{\tau_k} \sum_{t=1}^{\tau_k} \bar{v}_{i,t} + w_{i,k}^2 \quad (8)$$

$$i = 1, \dots, n; \quad k = 1, \dots, K$$

And measurement equations in macroscopic level are,

$$y_{i,k}^1 = \frac{1}{\lambda_{i,k}} Q_{i,k} \cdot e^{-\frac{\mu L_i}{V_{i,k}}} + \eta_{i,k}^1 \quad (9)$$

$$y_{i,k}^2 = V_{i,k} + \eta_{i,k}^2 \quad (10)$$

In fact, system equations (7) and (8) connect macroscopic states with microscopic states. k is a macroscopic time interval, (usually not equally long, from 5 to 30 minutes), which can be divided into τ_k much smaller intervals (which are usually equally long, such as 10 seconds). $Q_{i,k}$ is the number of vehicles crossing the handoff point, leaving link i and entering link $i+1$, during the interval $[t_k, t_{k+1}]$ or during time period k (in short, we call $Q_{i,k}$ the out flow of link i during time period k). $U_{i,t}$ is the microscopic out flow variable of link i in time interval t . $V_{i,k}$ represents the average traffic speed during time period k and $\bar{v}_{i,t}$ represents the average speed in time interval t , both of link i . Let T_k denote the length of time period k and let Δt denote the length of time interval t , both of which satisfy $\tau_k \cdot \Delta t = T_k$.

In measurement equation (9), L_i is the length of handoff link i and μ is a positive constant and $\lambda_{i,k}$ is a scale coefficient which presents how many percent in the running vehicles are using cell phones. The measurement model represented by (9) is designed according to following observations. First, the length of a call usually obeys an exponential distribution. Second, the longer a handoff link is, the fewer the valid handoff pair data can be caught. Third, the faster the vehicles run, the more the possible two continuous handoffs could happen. $y_{i,k}^1$ is the measured value of valid handoff pairs happened in link i . $y_{i,k}^2$ is the measured value of $V_{i,k}$. $\eta_{i,k}^1$, $\eta_{i,k}^2$ are measurement errors which are both zero-mean white noises.

In the lower lever, or microscopic lever, the state evolution is governed by following equations,

$$N_{i,t+1} = N_{i,t} + U_{i-1,t} - U_{i,t} \quad (11)$$

$$\bar{v}_{i,t+1} = \theta \bar{v}_{i,t+1}^{\text{int erm}} + (1-\theta)v^e(\rho_{i,t+1}^{\text{antic}}) + w_{i,t}^3 \quad (12)$$

where

$$\bar{v}_{i,t+1}^{\text{int erm}} = \begin{cases} (\bar{v}_{i-1,t} Q_{i-1,t} + \bar{v}_{i,t} (N_{i,t} - Q_{i,t})) / N_{i,t+1} & N_{i,t+1} \neq 0 \\ v_{\text{free}} & o.w. \end{cases} \quad (13)$$

and

$$\rho_{i,t+1}^{antic} = \sigma \rho_{i,t+1} + (1 - \sigma) \rho_{i+1,t+1} \quad (14)$$

Equation (11) expresses the evolution of $N_{i,t}$ based on the conservation of vehicles, where $U_{i,t}$ is the out flow of link i in time interval t . $\bar{v}_{i,t+1}^{interm}$ is the intermediate speed taking convection into account. Based on the assumption that drivers do not change their speed instantaneously due to inertia, it expresses the speed update if all vehicles would maintain their speed. $\rho_{i,t+1}^{antic}$ is called anticipated traffic density which drivers see at some distance in front of their vehicles, and $\rho_{i,t+1}$ can be computed via following equation

$$N_{i,t} = \rho_{i,t} L_i, \quad i = 1, 2, \dots, n \quad (15)$$

Here we assume all the links along the road have the same number of lanes. The coefficient $\sigma \in [0, 1]$ weighs how far ahead the drivers are looking in their anticipation. The non-linear function $v^e(\rho)$ expresses the average equilibrium speed corresponding to density ρ which can be computed according to the following empirical equation.

$$v^e(\rho) = \begin{cases} v_{free} \cdot e^{-0.5(\rho / \rho_{crit})^{3.5}} & \text{if } \rho \leq \rho_{crit} \\ v_{free} \cdot e^{-0.5(\rho - \rho_{crit})} & \text{otherwise} \end{cases} \quad (16)$$

where ρ_{crit} denotes the critical density.

To summarize, the right-hand side of speed state equation (12) is the sum of three items. The first item reflects the portion of vehicles keeping their current speed due to inertia. The second item reflects the portion where drivers aggressively adjust their speed to changing traffic conditions. $\theta \in [0, 1]$ is the weighing coefficient between the two items. The third item, noise $w_{i,t}^3$, reflects the unpredictable behavior of drivers and modeling errors.

The evolution of $U_{i,t}$ is dominated by sending and receiving functions [9]

$$U_{i,t} = \min(S_{i,t}, R_{i,t+1}) \quad (17)$$

Among the values of the above equation, the sending function is

$$S_{i,t} = \max\left(N_{i,t} \frac{v_{i,t} \Delta t}{L_i} + w_{i,t}^4, N_{i,t} \frac{v_{out, \min} \Delta t}{L_i}\right) \quad (18)$$

and the receiving function is

$$R_{i,t+1} = L_{i+1} / A_l + U_{i+1,t} - N_{i+1,t} \quad (19)$$

The sending function $S_{i,t}$ in (18) is a random variable expressing how many among the $N_{i,t}$ vehicles in link i at t are at a distance less than $v_{i,t} \Delta t$ from the handoff point between link i and link $i+1$. $v_{out, \min}$ denotes the minimum outflow speed. The receiving function (19) expresses the maximum

number of vehicles that are allowed to enter link $i+1$ at the next time instant $t+1$. The first item of the right-hand side of (19) actually denotes the maximum number of vehicles link $i+1$ can hold simultaneously. l is the number of lanes all long the road and A_l is the average length of vehicles plus a safe distance.

The state evolution of microscopic model described by (11) to (19) usually performs well on dense-sampling traffic estimation problems. The computing of sending and receiving functions is the core idea of this model. From (18) we can see that only when $v_{i,t} \Delta t < L_i$, the sending function makes sense. That needs Δt be very short because the handoff link length in urban area is usually from several hundred meters to less than 100 meters. So the microscopic model couldn't be used directly on cellular probe based traffic estimation. Hence, the idea of designing a two-level estimation model is coming out. The measurement all happens in macroscopic level and the two measurable values are the number of valid handoff pairs and the average speed derived from all handoff pair data during a not-very-short

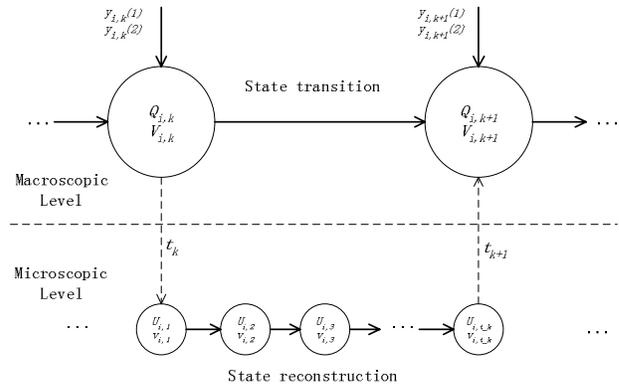


Fig. 3. State transition and reconstruction of two-level second-order traffic model

time period (for collecting enough data). The predictions of new macroscopic states are based on the computing of microscopic states according to (7) and (8). Upon obtaining the new measured value, the macroscopic states get updated finally. Then the system will switch to microscopic level again and the whole process continues shown as Figure 3.

IV. DESIGN OF PARTICLE FILTER

The core idea of Bayesian estimation is to construct the conditional PDF of the current state x_k , given all the available information: $p(x_k | Y_k)$. Here the available information at time step k is the set of measurements $Y_k = \{y_j, j = 1, \dots, k\}$. In principle, this PDF may be obtained recursively in two stages: prediction and update [6][7]. Suppose that the required PDF

$p(x_{k-1} | Y_{k-1})$ is available.

Prediction:

$$p(x_k | Y_{k-1}) = \int p(x_k | x_{k-1}) p(x_{k-1} | Y_{k-1}) dx_{k-1} \quad (20)$$

Updating:

$$p(x_k | Y_k) = \frac{p(y_k | x_k) p(x_k | Y_{k-1})}{p(y_k | Y_{k-1})} \quad (21)$$

where $p(y_k | Y_{k-1})$ is a normalized constant.

We assume that the PDFs of independent noise term in generic system equation (2) and measurement equation (3) are $p(w_k)$ and $p(\eta_k)$. So the probability model of system evolution $p(x_k | x_{k-1})$, which is a Markov model, can be defined by the system equation and the known statistics of w_k .

$$p(x_k | x_{k-1}) = \int \delta(x_k - f_{k-1}(x_{k-1}, w_{k-1})) p(w_{k-1}) dw_{k-1} \quad (22)$$

Also, $p(y_k | x_k)$ is defined by the measurement equation and the known statistics of η_k .

$$p(y_k | x_k) = \int \delta(y_k - h_k(x_k, \eta_k)) p(\eta_k) d\eta_k \quad (23)$$

where $\delta(\cdot)$ is the Dirac delta function.

Usually, there is no simple analytical solution for $p(x_k | Y_k)$ due to the difficulty of computing complex high-dimension integrals in (20), (22) and (23). The particle filter technique can provide an approximate solution to the discrete-time recursive updating of the posterior PDF $p(x_k | Y_k)$ by the empirical histogram corresponding to a collection of M particles $\{x_k^{(l)}, l = 1, \dots, M\}$.

$$\hat{p}(x_k | Y_k) \approx \sum_{l=1}^M \bar{q}_k^{(l)} \delta(x_k - x_k^{(l)}) \quad (24)$$

where $\delta(\cdot)$ is Dirac delta function and M is the number of random samples or particles. The particles $x_k^{(l)}$ and weights $\bar{q}_k^{(l)}$ are recursively updated as follows,

$$x_k^{(l)} \sim p(x_k | x_{k-1}^{(l)}), \quad l = 1, \dots, M \quad (25)$$

$$\bar{q}_k^{(l)} = \frac{p(y_k | x_k^{(l)}) \bar{q}_{k-1}^{(l)}}{\sum_{j=1}^M p(y_k | x_k^{(j)}) \bar{q}_{k-1}^{(j)}}, \quad l = 1, \dots, M \quad (26)$$

The implementation of the particle filter is described as follows.

Step1: Initialization

For $l = 1, \dots, M$, sample $x_0^{(l)} \sim p(x_0)$,

$\bar{q}_0^{(l)} = 1/M$ and set $k = 1$.

Step2: Prediction

For $l = 1, \dots, M$, sample $x_k^{(l)} \sim p(x_k | x_{k-1}^{(l)})$

Step3: Importance evaluation

On receiving a new measurement, compute the normalized weights,

$$\bar{q}_k^{(l)} = (p(y_k | x_k) \bar{q}_{k-1}^{(l)}) / \sum_{j=1}^M p(y_k | x_k^{(j)}) \bar{q}_{k-1}^{(j)},$$

$l = 1, \dots, M$, where $p(y_k | x_k^{(l)})$ can be described by observation equation (5) for first-order model or (9),(10) for second-order model.

Step4: Selection

Multiple / Suppress M particles $\{x_k^{(l)}\}$ according to their importance weights in order to obtain new M unweighted particles still having an approximate distribution of $p(x_k | Y_k)$. Here we choose residual resampling algorithm[7].

Step5: Output

According to (18), we can compute the approximate posterior distribution based on the samples generated on previous step 3. Also the posterior mean and covariance can be computed as follows.

$$\hat{x}_k = E(x_l | Y_k) = \frac{1}{M} \sum_{l=1}^M x_k^{(l)} \quad (27)$$

$$V(x_k | Y_k) = \frac{1}{M-1} \sum_{l=1}^M (x_k^{(l)} - \hat{x}_k)(x_k^{(l)} - \hat{x}_k)^T \quad (28)$$

Step6: Let $k = k + 1$ and return to step 2.

V. NUMERICAL EXPERIMENTS

To check the capability of particle filters to make the traffic state estimation, we use two testing data sets which are collected from typical real mobile network. The first is a 15 hour long (8:00am to 11:00pm) data set based on a segment of urban expressway including three handoff links, where the macroscopic time interval is 30 minutes and $L = (0.558, 0.411, 0.242)^T$, [km]. The second data set is 24 hours long based on another 3-handoff-link expressway segment, where the macroscopic time interval is 5 minutes and $L = (0.284, 0.881, 0.272)^T$, [km].

Figure 4 shows the average speed estimation result on dataset one. We can see that the first-order model tracks the noisy observation value quickly and it is hardly to tell apart them in most of the time. The output of the second-order model is different, always keeping a gap with observation value, but they follow similar changing tendency at most time periods except in the situation of link 2.

Figure 5 shows the outflow estimation result of the second-order model. The curve in black color is the derived outflow value according to equation (9). We can see that the evolving tendencies of the estimated and derived value are consistent, but the estimated flow values are 50% to 100% greater than derived value in link 3 and 4. The accumulated error during every 30-minutes-long process of microscopic

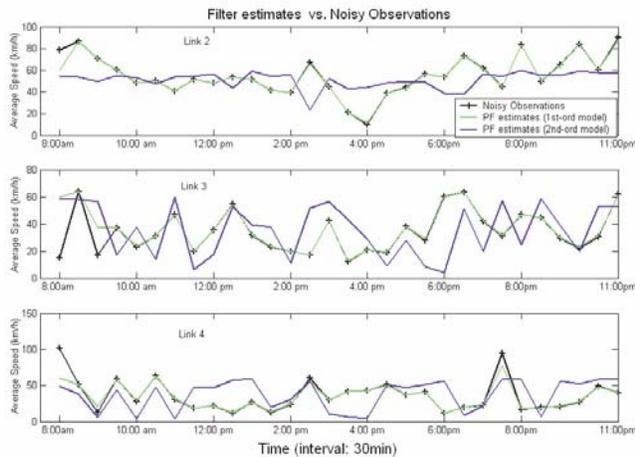


Fig. 4. Speed estimation result of two different particle filters

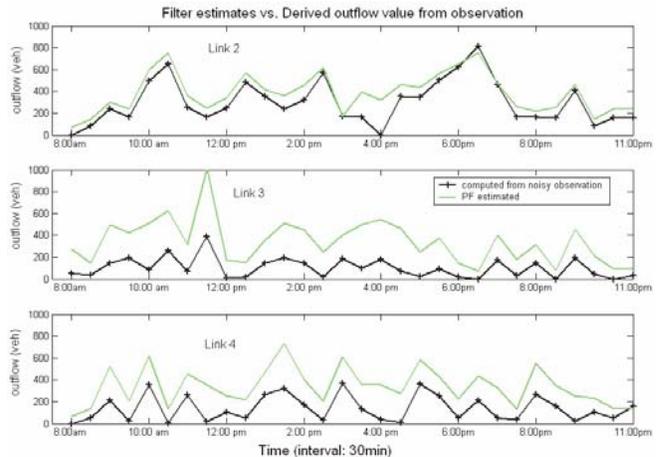


Fig. 5. Flow estimation result of particle filter for the second-order traffic model

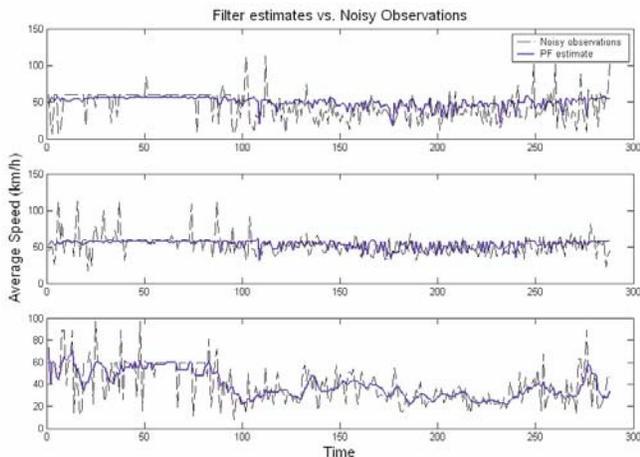


Fig. 6. Speed estimation result on data set 2. (Time interval: 5 minutes)

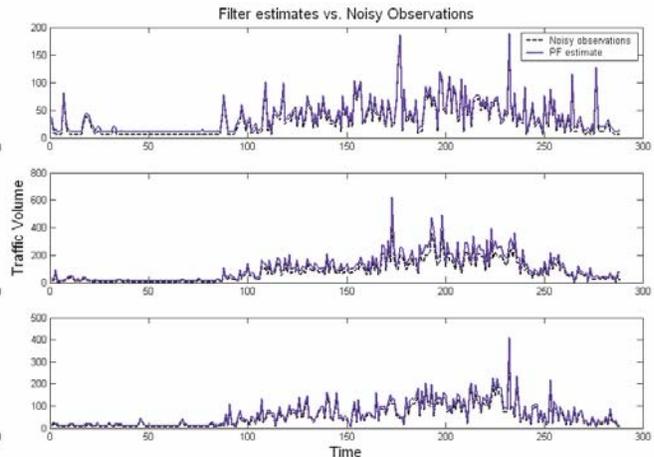


Fig. 7. Flow estimation result on data set 2. (Time interval: 5 minutes)

$$V_{free} = 60[\text{km/h}], V(w_{i,k})=200[\text{km/h}]^2, \text{ and } V(\eta_{i,k})=4[\text{km/h}]^2. \rho_{crit} = 120 [\text{veh/km}], \Delta t = 10 [\text{s}]. M=200$$

state evolution may be one reason and the lack of field data to calibrate the parameters in (9) is the another reason.

Figure 6 and 7 show the state estimation result of the second-order model based on dataset two. Because the macroscopic time interval here decreases to 5 minutes, the number of valid observed date in each interval also decreases. Here we use some date pre-processing method to interpolate missing data. It can be seen that the volume estimation tracks the observation date well but the speed estimation still has fairly large error.

VI. CONCLUSION

The aim of this paper is to introduce an approach of traffic state estimation using handoff data of cell phones in floating vehicles. Two models for traffic state estimation were developed based on Bayesian estimation theory. The two-level structure of the second-order model is designed to adapt the fact of cellular handoff probe as one kind of sparse-sampling traffic estimation. Both models were implemented by particle filters. Numerical experiments show the primal comparison result between estimated state and noisy observations. The accuracy of particle filter needs to be

improved in the future research by parameter adjustments and calibrations incorporating other data sources such as loop data.

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