Network localization and identification of rigid components from bearing measurements

Extended abstract

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I. INTRODUCTION

Sensor network localization is a problem which arises in many circumstances when sensors are deployed in an environment. We focus specifically on the case when sensors are able to take bearing measurements and cannot measure distances. This problem occurs, for example, in robotic networks [1]: if each robot is equipped with a set of antennae, it may be able to tell which direction another robot is located with respect to itself but not how far away it is. This problem is also seen in camera networks since standard cameras can only determine the relative bearings between points, but not depth. A related problem arises in computer vision as the "structure from motion" problem, where camera measurements of points in the world are used to reconstruct the network consisting of both points and cameras [2], [3].

We investigate the circumstances under which a network can be localized from only such bearing measurements. We consider the problem under two different situations. In the first – shown in Figure 1a – all nodes in the network have access to a global coordinate frame. In this case, we review previous results on localization. In addition, we present a method for identifying maximal rigid components of the network.

In the second case – depicted in Figure 1b – no global coordinate frame is available and nodes are only able to make relative bearing measurements. For this case, localization is more difficult. We propose a "triangular" constraint for this situation that is linear and allows for exact solutions to triangulated networks. We then extend this constraint to general networks and propose two efficient optimization procedures. This approach can also be extended to 3D networks under certain circumstances, and we evaluate our approach on several datasets to show its applicability.

II. WITH A GLOBAL COORDINATE FRAME

Given a network of n nodes, we first consider the setup where each node is capable of measuring its bearing with respect to a global coordinate frame, as shown in Figure 1a. For an embedding problem in $\mathbb{R}^d$, we write the location of node i as $x_i = \begin{bmatrix} x_i^1 & \dots & x_i^d \end{bmatrix}$. Given an embedding $x = \begin{bmatrix} x_1 & \dots & x_n \end{bmatrix}^T \in \mathbb{R}^{dn}$ satisfying all angular constraints, any translation or scaling of x is also a valid embedding since these transformations maintain the global angles between nodes.

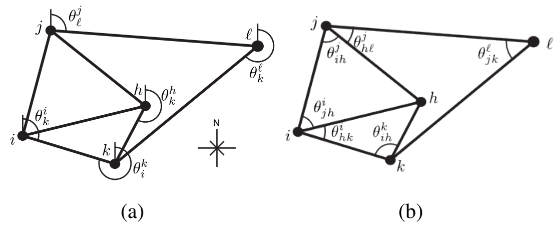

Fig. 1: **(a)** A directed network of nodes with global angle measurements; when a global orientation reference is available, angles can be measured with respect to this coordinate system. **(b)** A directed network of nodes with relative angle measurements. The component formed by nodes h, i, j and k is a rigid set of triangular constraints while node ℓ is connected rigidly to this component.

In this case, the localization problem has been previously studied by Brand [4]. In particular, it was shown that the set of all bearing constraints can be written as a linear system $Ax = 0$, for which a least-squares solution can be easily found.

If a single, unique solution $\mathbf{x}$ exists for an embedding problem (up to these invariant transformations), we say that this solution is *rigid*. Using the set of solutions to the linear system $Ax = 0$, it is possible to identify the set of maximally-rigid components of the network. In particular, let N be a matrix such that the columns of N span the null space of the matrix A. Then, for any weight vector w, we have $A(Nw) = 0$ and thus and $x = Nw$ is a solution to our system. We show how this null space matrix N can be used to identify all maximal rigid components.

III. WITHOUT A GLOBAL COORDINATE FRAME

We also consider the setup where each node is capable of measuring its relative bearing to other nodes within the network but where no global coordinate frame is known. Equivalently, each node is able to measure the angle between other nodes relative to its own position. This setup is depicted in Figure 1b. The goal is to estimate the overall layout of the network, up to a similarity transformation, based on this bearing information.

The 2D layout of the network of n nodes is then represented by a vector $x = \begin{bmatrix} x_1 & x_2 & \dots & x_n \end{bmatrix}^T \in \mathbb{C}^n$. The

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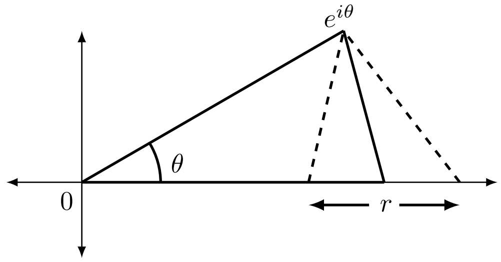

Fig. 2: An angle constraint can be written as a triangle constraint with one unknown parameter. We parameterize this constraint as a triangle with an unknown side length, $r \in \mathbb{R}_+$.

Cartesian coordinates of a node $x_i \in \mathbb{C}$ are given by the real and imaginary parts of the number x_i, respectively.

IV. LINEAR CONSTRAINT FOR TRIANGLES

Let i be a node which measures the angle between two other nodes j and k, denoted as θ^i_{jk}. If three nodes all observe each other, resulting in three angle measurements, then the combination of all three constraints forms a *triangular constraint*. This triangular constraint is satisfied by any triangle in the plane such that the three internal angles are the same as those specifed by the three bearing constraints. We show how such triangular constraints are, in fact, *linear*. The resulting system of linear equations can be easily solved, resulting in globally-optimal solutions for triangulated networks.

V. APPLICATION TO NON-TRIANGULAR PROBLEMS

To deal with non-triangular constraints, we consider an angle constraint as a triangle constraint with one unknown angle. In this way, an angle constraint can be written as a triangle constraint *with an unknown parameter*. This is shown in Figure 2. Given a specified angle θ, the parameter r controls the radius of one side of the triangle. A network with arbitrary bearing measurments can be represented in this way, as a set of parameterized linear constraints. We write this system of equations as $A(r)x = 0$.

VI. OPTIMIZATION

We propose two different methods which rely on local optimization to solve the system of parameterized linear constraints. The first is a simple alternating method. Beginning with an initialization of r, the problem is linear and can be solved optimally for x. Similarly, for a fixed value of x, the optimal value of r can be easily found.

The second approach is based on the observation that for a fixed value of r the optimal cost is given by the second-smallest eigenvalue of a certain matrix. Indeed, the cost function can be regarded as a function maps the parameter vector r to the second-smallest eigenvalue of a parameterized matrix. A local optimization can be performed using the derivative of this function [5] in a gradient descent scheme.

VII. APPLICATION TO $\mathbb{R}^3$ WITH GRAVITY

The problems discussed so far are not directly generalizable to 3-dimensional space. However, in real-world applications, it may be possible to determine the vertical axis in the world even if the full orientation of the sensors is not known. For example, photographs tend to be taken such that they are aligned vertically or a vertical axis can be determined by lines in the image such as the sides of a building, or in a sensor network each node may be equipped with an accelerometer that can be used to determine the direction of gravity. In any case, if a vertical axis can be determined then we show how a 3D reconstruction can be decomposed into two parts. First, all angles are measured orthogonal to the known vertical direction (i.e., projected onto the 2D ground plane). The 2D layout of the network, as if viewed from above, can be estimated using the methods presented previously. Then, given an estimated 2D layout of the network, we show how the vertical positions of the nodes can be written as a linear system and easily found.

VIII. EXPERIMENTS

We evaluate our algorithms on several datasets. First, we generate synthetic datasets to determine the succeptibility of our methods to noise and find that they perform well. We also apply our approach to “structure from motion” datasets, where the network consists of both cameras and scene points and the goal is to reconstruct the full layout of all nodes. This done in 2D using synthetic datasets, as well as in 3D using a real dataset of a toy dinosaur.

REFERENCES

[1] G. Mao, B. Fidan, and B. Anderson, “Wireless sensor network localization techniques,” *Computer networks*, vol. 51, no. 10, pp. 2529–2553, 2007.

[2] S. Agarwal, N. Snavely, S. M. Seitz, and R. Szeliski, “Bundle adjustment in the large,” in *Computer Vision–ECCV 2010*. Springer, 2010, pp. 29–42.

[3] D. Crandall, A. Owens, N. Snavely, and D. Huttenlocher, “Discrete-continuous optimization for large-scale structure from motion,” in *Computer Vision and Pattern Recognition (CVPR), 2011 IEEE Conference on*. IEEE, 2011, pp. 3001–3008.

[4] M. Brand, M. Antone, and S. Teller, “Spectral solution of large-scale extrinsic camera calibration as a graph embedding problem,” *European Conference on Computer Vision*, pp. 262–273, 2004.

[5] J. H. Gallier, *Geometric methods and applications: for computer science and engineering, second edition*. Springer, 2011, vol. 38.