# Factor-based Underwater Cooperative Localization over a Low-Bandwidth Communication Channel

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## I. Introduction

Autonomous underwater vehicles (AUVs) typically integrate body-frame velocities, attitude, and pressure depth to compute a dead-reckoned (DR) navigation solution. Errors in $xy$ horizontal position estimates grow unbounded in time without regular access to an absolute position reference (GPS is only available at the surface). Bounded error navigation can be achieved with the aid of fixed acoustic beacon systems such as long-baseline (LBL). While these systems can accurately localize an AUV, they do not scale well to large vehicle networks and can be expensive to deploy.

Synchronous-clock acoustic hardware allows communicating vehicles to observe their relative range via the one-way-travel-time (OWTT) of acoustic broadcasts [1]. OWTTs provide a relative range measurement between the transmitting vehicle pose at the time-of-launch (TOL) and the receiving vehicle pose at the time-of-arrival (TOA). The acoustic channel, however, is broadcast and unacknowledged, provides very low bandwidth (typically less than 100 bps), and displays low reception rates (often less than 50%).

We propose a cooperative localization framework in which underwater vehicles act as mobile navigation beacons. Each vehicle tracks a joint distribution over a set of its own delayed-states (historic poses along its trajectory) and a set of delayed-states belonging to its team members. We employ a factor graph representation of the joint distribution in order efficiently share information between vehicles and fuse them in a consistent way. Moreover, this representation allows each vehicle to partially reconstruct the full joint distribution over all vehicle delayed-states without enforcing a specific communication topology.

## II. Factor-based cooperative localization

Factor graph estimation frameworks [2] have become a popular tool in cooperative localization [3–5]. The underlying structure of the factor graph consists of information local to each vehicle and information due to relative vehicle measurements.

In the single vehicle setting, the factor graph approach is a smoothing algorithm that estimates the entire trajectory of the vehicle. A factor graph is a bipartite graph with pose (variable) nodes and factor (measurement) nodes representing the joint distribution over the unknown poses. The $i$th vehicle graph represents the joint distribution over its $N$ poses, $\mathbf{X}_i = [\mathbf{x}_1, \ldots, \mathbf{x}_N]$, as

$$p(\mathbf{X}_i) \propto p(\mathbf{x}_1) \prod_i p(\mathbf{z}_{\text{odo}_i} | \mathbf{x}_i, \mathbf{x}_{i-1}) \prod_j p(\mathbf{z}_{\text{prior}_j} | \mathbf{x}_j), \quad (1)$$

where we assume each vehicle has access to its initial belief $p(\mathbf{x}_1)$. The graph structure is a chain as we only consider unary 'prior' factors, $\mathbf{z}_{\text{prior}}$, (e.g., GPS when available at the surface) and pairwise sequential 'odometry' factors, $\mathbf{z}_{\text{odo}}$, (e.g., integrated velocity).

For convenience, we define a 'link', $\mathcal{L}_i$, associated with the $i$th pose node, $\mathbf{x}_i$, as a 2-tuple containing the odometry factor to the previous pose node and a prior factor. The local chain is the set of links which represent the vehicle trajectory corresponding to (1), $\mathcal{C}_{\text{local}} = \{\mathcal{L}_i\}_{i=1}^N$. In Fig. 1, each row of pose nodes in the graph represents a local chain.

We can construct the factor graph over the entire $M$ vehicle network (i.e., all vehicle poses), $\{\mathbf{X}_1, \ldots, \mathbf{X}_M\}$,

$$p(\mathbf{X}_1, \ldots, \mathbf{X}_M) \propto \prod_i \underbrace{p(\mathbf{X}_i)}_{\mathcal{C}_{\text{local}_i}} \prod_k \underbrace{p(\mathbf{z}_k | \mathbf{x}_{i_k}, \mathbf{x}_{j_k})}_{\text{relative factors}}, \quad (2)$$

where each $\mathbf{z}_k$ represents a relative vehicle constraint between poses on vehicles $i_k$ and $j_k$. Here, $\mathbf{z}_k$ is a OWTT range constraint between a transmitting vehicle's TOL pose and a receiving vehicle's TOA pose. The factor graph for a three vehicle network is illustrated in Fig. 1.

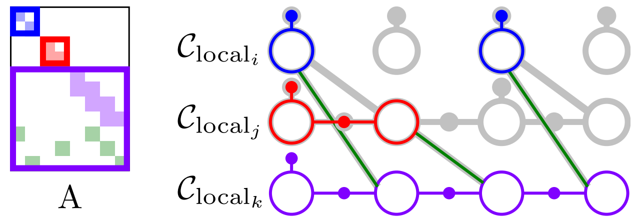


Fig. 1: Example factor graph estimation framework and corresponding measurement Jacobian, A. Each row of pose nodes (large circles) represents a single vehicle. The full (centralized) graph is highlighted in gray, while the reconstruction on board the third (purple) vehicle is fully colored.

The gold-standard would be to compute the maximum a posteriori (MAP) estimate for each vehicle in a centralized estimator as

$$\mathbf{X}_i^* = \text{argmin}_{\{\mathbf{X}_1, \ldots, \mathbf{X}_M\}} - \log p(\mathbf{X}_1, \ldots, \mathbf{X}_M). \quad (3)$$

For Gaussian noise models, the MAP estimate results in a nonlinear least-squares problem with linear subproblem

$$\min_{\mathbf{X}} \left\| \mathrm{A}\mathbf{X} - \mathbf{b} \right\|^2, \quad (4)$$

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where A is the measurement Jacobian weighted by the square root information [2].

The full joint distribution (2) consists of a product of each vehicle's local chain and the relative vehicle factors. Therefore, in order to construct (and perform inference on) the full factor graph, the $i$th vehicle must have access to the set of local factors from all other vehicles, $\{\mathcal{C}_{\text{local}_j}\}_{j\neq i}$, and the set of all relative vehicle factors. Sharing this information, however, is nontrivial due to the limitations of the acoustic communication channel.

We propose two methods to reliably broadcast each vehicles local chain throughout the network as the set of individual factors. First, we exploit composition/decomposition operations over odometry factors that are tolerant to communication dropout. Second, we employ approximate marginalization [6] to accurately summarize the full set of factors by a smaller set of approximate factors. Within the resulting framework, each vehicle regularly broadcasts a fixed-bandwidth data packet containing a composed odometry factor to its current position and several previously broadcast prior factors. This broadcast information allows receiving vehicles to reconstruct an informative portion of the transmitter's local chain.

Each vehicle can then reconstruct a portion of full factor graph including the set of reconstructed chains, its own local chain, and the set of relative vehicle measurements it has observed locally (a subset of all relative observations). Inference is performed as a batch procedure on this reconstructed factor graph as in (3).

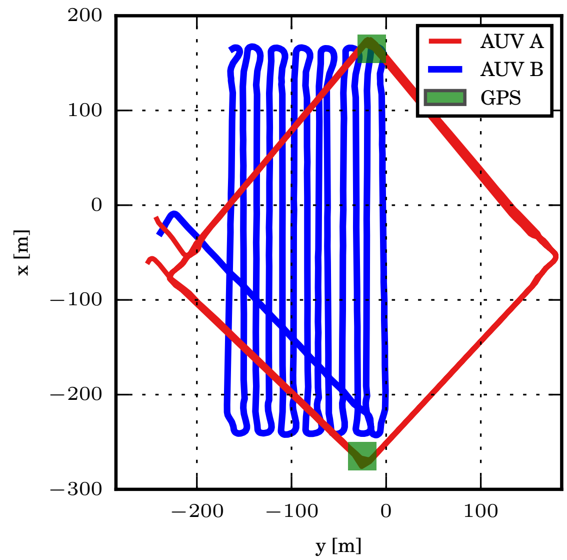
## III. Field Trials

For validation, we fielded two Ocean-Server Inc. Iver2 AUVs, termed AUV-A and AUV-B, and a topside support ship. Results over a single trial are summarized in Fig. 2. A topside vehicle with constant GPS access supported AUV-A (with intermittent GPS) and AUV-B (see Fig. 2a). AUV-A followed a large diamond over AUV-B's lawnmower survey while the topside vehicle drifted above the survey area.

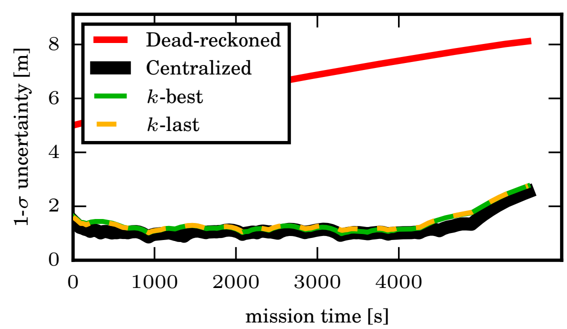
Fig. 2b plots AUV B's position estimate uncertainty for the centralized estimator as well as the dead-reckoned and decentralized estimators. Although we only compare to the centralized estimator for AUV-B, note that all vehicles compute a local reconstruction of the centralized estimator. In this case, the centralized estimator used ranges between all three vehicles. Our method only used ranges between the local platform and the other vehicles, but still produces an accurate estimate as evidenced by the resulting uncertainty.

## IV. Conclusions

Accurate localization extends the capacity of AUVs to perform ocean science. OWTT underwater cooperative localization promises improved navigation for AUVs over larger area and time scales without additional infrastructure. We exploited the structure of the composition operation and an accurate approximation of the local chain to robustly share locally observed sensor data across a fragile communication channel. Compelling avenues for future work include sharing global information throughout the network, i.e., information beyond the local chain.




(a) Relative paths (1.55 h), AUV-A, topside (not shown) have GPS.




(b) AUV-B's estimate uncertainty.

Fig. 2: Summary of field trial and performance comparison. (b) plots the smoothed uncertainty in each AUV-B pose computed as the fourth root of the determinant of the pose marginal covariance.

## References


[1] R. M. Eustice, H. Singh, and L. L. Whitcomb, "Synchronous-clock one-way-travel-time acoustic navigation for underwater vehicles," *J. Field Robot.*, vol. 28, no. 1, pp. 121–136, 2011.

[2] F. Dellaert and M. Kaess, "Square root SAM," *Int. J. Robot. Res.*, vol. 25, no. 12, pp. 1181–1204, 2006.

[3] M. F. Fallon, G. Papadopoulis, and J. J. Leonard, "A measurement distribution framework for cooperative navigation using multiple AUVs," in *Proc. IEEE Int. Conf. Robot. and Automation*, Anchorage, AK, May 2010, pp. 4256–4263.

[4] A. Cunningham, V. Indelman, and F. Dellaert, "DDF-SAM 2.0: Consistent distributed smoothing and mapping," in *Proc. IEEE Int. Conf. Robot. and Automation*, Karlsruhe, Germany, May 2013, pp. 5220–5227.

[5] L. Paull, M. Seto, and J. J. Leonard, "Decentralized cooperative trajectory estimation for autonomous underwater vehicles," in *Proc. IEEE/RSJ Int. Conf. Intell. Robots and Syst.*, Chicago, IL, Sep. 2014, pp. 184—191.

[6] M. Mazuran, G. D. Tipaldi, L. Spinello, and W. Burgard, "Nonlinear graph sparsification for SLAM," in *Proc. Robot.: Sci. & Syst. Conf.*, Berkeley, CA, Jun. 2014.